A Normative Argument Against Explosion

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Abstract. One strategy for defending paraconsistent logics involves raising 'normative arguments' against the inference rule *explosion*. Florian Steinberger systematically criticises a wide variety of formulations of such arguments. I argue that, for one such formulation, Steinberger's criticisms fail. I then sketch an argument, available to those who deny dialetheism, in defence of the formulation in question.

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1. Normative arguments against explosion

Paraconsistent logics reject the following inference rule.

> *Explosion* (EXP): An inconsistent set of premises entails *Q*, where *Q* is any proposition whatsoever.

Such logics are typically packaged with dialetheism—the view that there are true contradictions—but need not be. The dialetheist may endorse EXP if she is willing to countenance the truth of *every* proposition;[1] and the non-dialetheist may have independent reasons for denying EXP.[2]

[1] For an example of *trivialism*, see e.g. Azzouni 2013.

[2] For example, that conclusions ought to relevant to premises; see e.g. Anderson and Belnap 1975.

A popular strategy for defending paraconsistent logics involves raising normative considerations against EXP. Consider this argument.

> One ought to believe the logical consequences of one's beliefs. But, in fact, most people have inconsistent beliefs. Supposing EXP (for *reductio*), it follows that most people ought to believe *Q*, where *Q* is any proposition whatsoever. But, for any given person, there is some proposition that she ought *not* to believe. It follows from the preceding that, for some *R*, she ought to believe *R* and ought not to believe *R*. This is a contradiction. Thus, by *reductio ad absurdum*, EXP is not a valid rule of inference.

Call this a *normative argument* (*against explosion*).

Let the *bridge principle* in such an argument be the premise that connects logical consequence with normative epistemic requirements. The bridge principle in the above is:

B_1: One ought to believe the logical consequences of one's beliefs.

There are at least two serious objections to B_1. The first is due to Harman 1984. Suppose that my current beliefs have an absurdity as a logical consequence. Then, given B_1, I ought to believe the absurdity. But that consequence is false: I ought rather to revise one of my antecedent beliefs. The second is due to Broome 1999. Each proposition is a logical consequence of itself. So, given B_1, one ought to believe everything that one does *in fact* believe. But that consequence is false: sometimes one forms beliefs without, say, taking due care as to whether the belief is true.

Building on work by MacFarlane (2004), Florian Steinberger (2016) investigates whether there is a successful formulation of the normative argument against explosion by systematically considering alternative bridge principles. He argues not, concluding that 'there is no successful way of reformulating the normative argument' (p. 387).

Steinberger's criticisms are largely successful, but fail for one particular bridge principle. I introduce the bridge principle in §2, respond to Steinberger's criticisms in §3, and argue that the

bridge principle can be independently justified in §4. In §5, I sketch an argument, available to non-dialetheists, in defence of the corresponding formulation of the normative argument.

To be explicit, I do not show herein that the relevant formulation of the normative argument is *successful*: I doubt such a conclusion could be established in a single article. But, nonetheless, I provide a clear strategy for developing the argument into a potentially compelling case against EXP, at least in the eyes of the non-dialetheist. Given that (i) normative arguments have recently received substantial criticism (especially in Steinberger 2016), and (ii) most non-dialetheists endorse EXP, that strategy is, I suggest, of no little importance.

2. The B_2-formulation

The bridge principle in question is this:

B_2: If $P_1, \ldots, P_n \vDash Q$, then if *S* believes all the P_i, *S* has (defeasible) reason to believe *Q*.

(Following MacFarlane, Steinberger calls it (Cr+).) Using B_2, the normative argument is formulated as follows (Steinberger 2016: 389, 401, 403). For some ordinary person *S*, and inconsistent set of propositions Φ:

(a) EXP is valid. [Hypothesis, for *reductio*.]

(b) *S* believes each member of Φ. [Premise]

(c) If $P_1, \ldots, P_n \vDash Q$, then if *S* believes all the P_i, *S* has (defeasible) reason to believe *Q*. [B_2]

(d) Even if *S*'s set of beliefs is inconsistent and any proposition whatsoever is entailed by it (courtesy of EXP), there are propositions that *S* has no reason whatsoever to believe. [Premise]

(e) *S* has no reason whatsoever to believe *R*. [From (d).]

(f) $\Phi \vDash R$. [From (a).]

(g) *S* has (defeasible) reason to believe *R*. [From (b), (c) and (f).]

(h) Contradiction. [From (e) and (g).]

(i) EXP is invalid. [From (a)–(e) by *reductio*.]

Call this the *B_2-formulation* of the normative argument.

Note that, throughout this paper, I take it as common ground that (b) is true of many ordinary people.[3] Moreover, as Steinberger's principal criticisms concern (c), I provisionally assume (d)—although I drop this assumption in §5.

As Steinberger notes (2016: 402), B_2 avoids both Harman's and Broome's criticisms: it does not yield any consequences about what one *ought* to believe at all. However, Steinberger raises two alternative objections to B_2: first, that it is implausibly strong; second, that it is implausibly weak.

3. Steinberger's criticisms of B_2

First, Steinberger writes that, if B_2 is philosophically tenable, then

> I *do* have a reason (albeit a defeasible one) for believing any logical consequence of my beliefs. And so, saying that I have no reason whatsoever to believe *P* given my background beliefs and my evidence is to say that I *could not possibly believe* (not merely that I do not have any reason to believe) any set of propositions that entails *P* (otherwise, by [B_2], I *would* have a reason to believe *P*). But that seems implausibly strong. (2016: 403)

I find this objection puzzling. Reconstructed, the objection is that B_2 implies (1):

(1) If *S* could believe a set of propositions that entails *P*, then *S* has a (defeasible) reason to believe *P*.

[3] See Steinberger 2016: 392–395.

and that (1) entails the 'implausibly strong' conclusion that 'saying that I have no reason whatsoever to believe *P* is to say that I *could not possibly believe* any set of propositions that entails *P*'. Thus, Steinberger concludes, B_2 is implausibly strong.

However, B_2 does *not* imply (1): B_2 underpins an inference from what *S does* believe to what her (defeasible) reasons are; it does not permit any inferences at all from what *S could* believe. To obtain (1) from B_2, Steinberger requires an auxiliary premise:

(2) If *S* could believe a set of propositions that entails *P*, then *S* does believe a set of propositions that entails *P*.

However, Steinberger is not entitled to assume (2). The most plausible defence of (2) appeals to two claims: that *S* has inconsistent beliefs; and that EXP is valid. (These two claims jointly imply the truth of (2)'s consequent, and thus of (2).) But Steinberger is not entitled to assume that EXP is valid, as this is precisely what is at issue.

The only consequence of B_2 in the vicinity of (1) is:

(3) If *S* could believe a set of propositions that entails *P*, then *S* could have (defeasible) reason to believe *P*.

But (3) is unproblematic, and does not imply Steinberger's 'implausibly strong' conclusion.

The second objection is that B_2 is implausibly weak.

> [B_2] in no way requires of me that I should revise my beliefs in light of their consequences. Surely, though, logical coherence *does* demand that I modify my belief set so as to avoid blatant inconsistencies. (Steinberger 2016: 404)

This objection presupposes that B_2 is intended to capture normative requirements *of logic*. However, this presupposition is *not* required for the B_2 formulation of the normative argument to go through: on

the natural reading of (d)—on which 'no reason whatsoever' implies that there is no reason logical *or otherwise*—there is no equivocation. I return to this point in the next section.

But even if B_2 *is* intended to capture normative requirements of logic, the objection fails. B_2 makes no claim of completeness: it is compatible with there being *other* normative requirements of logic. So the objection can be overcome by supplementing B_2 with an additional principle.[4]

Now, B_2 is limited because it only generates reasons *to believe*. Modifying a belief set to avoid inconsistencies, however, can require *ceasing* to believe. Thus, B_2 needs to be supplemented with a principle that generates reasons *not* to believe. For example:

B_3: If $P_1, \ldots, P_n \vDash Q$, then if S believes all the P_i, S has (defeasible) reason *not* to believe $\sim Q$.

Suppose, then, that I believe P and Q, and learn that $P \leftrightarrow \sim Q$. Given B_2, I have (defeasible) reason to believe each of $\sim P$, $\sim Q$, and $\sim(P \leftrightarrow \sim Q)$. Given B_3, I have (defeasible) reason *not* to believe each of P, Q, and $P \leftrightarrow \sim Q$. These reasons do not themselves tell me *which* belief(s) I ought to revise, but merely that I have reason to revise my belief set in one (or more) of the prescribed ways.

What I *ought* to do is determined by the relative strengths of my *other* reasons. If I have better reasons to believe Q and $P \leftrightarrow \sim Q$ than to believe P, then I may have overriding reason to revise the belief that P: I *ought* to cease believing that P and form the belief that $\sim P$. If I have equal reason to believe Q and P, and a better reason to believe $P \leftrightarrow \sim Q$, then I may have overriding reason to cease believing Q and P (but not to form any beliefs).

So, although B_2 *on its own* might be implausibly weak, it can be supplemented with B_3. Together, B_2 and B_3 generate reasons to revise an inconsistent belief set, thereby avoiding 'blatant inconsistencies'. They do *not* say *how* the belief set *ought* to be revised. But this is at it should be: in general, logic alone cannot find the false proposition in an inconsistent set.

So, jointly, B_2 and B_3 plausibly capture the logical norm to avoid 'blatant inconsistencies'. Thus, Steinberger's second objection fails.

[4] Steinberger recognises this: 'The solution, therefore, cannot lie with [B_2] (at least not taken on [its] own)' (2016: 404).

4. An independent justification of B_2

As noted above, however, the B_2-formulation of the normative argument goes through *regardless* of whether B_2 captures normative requirements *of logic*. To obtain a contradiction, B_2 need merely state that, in the relevant circumstances, *S* has a (defeasible) reason *of some kind* to believe *Q*. Call that the *neutral reading* of B_2.

On the neutral reading, B_2 can be *independently* justified. Consider the following principles:

P_1: *S* is entitled to take the contents of *S*'s beliefs as evidence.

P_2: If *P* is a logical consequence of *S*'s evidence, then *S* has (defeasible) reason to believe *P*.

Together, P_1 and P_2 imply that *S* is entitled to take herself to have (defeasible) reason to believe the logical consequences of her beliefs, which plausibly implies B_2. And, importantly, P_1 and P_2 can be independently justified.

For example, Timothy Williamson argues extensively for P_1 (2007: ch. 7). Building on the view that evidence is propositional, he argues: first, that the contents of one's knowledge constitutes one's evidence; and, second, that one is entitled to take one's beliefs as knowledge. Williamson puts his account to substantial philosophical work, explaining how thought experiments generate new knowledge and constructing a strategy for responding to any given form of scepticism. So there are established grounds for accepting P_1.

Consider P_2. It should be read as *neutral* as to whether the (defeasible) reason is logical or otherwise. In addition, note that P_2 states that *S* has a (defeasible) reason to believe the logical consequences of the propositions *that constitute her evidence*, not of the propositions *for which she has evidence*. For example, suppose that Sara is on a jury. Her evidence consists of two propositions: *that Jan was seen walking suspiciously towards Mike*; and *that Mike's bag was subsequently stolen*. Then, according to P_2, Sara has (defeasible) reason to believe *that Jan was seen walking suspiciously towards the subsequent victim of a bag-theft*. But P_2 is *silent* about whether Sara has (defeasible)

reason to believe, say, *that Jan stole Mike's bag*: the latter is not a logical consequence of Sara's evidence.

The justification of P_2 resides in the fact that evidence-based disciplines—the sciences, law, perhaps philosophy, etc.—go so far beyond it. In such disciplines, evidence is typically taken to *confer* (defeasible) reason to believe a proposition that the evidence *merely* supports. But it would be difficult to maintain such a claim while *denying* that we have (defeasible) reason to believe the logical consequences of our evidence. Our use of evidence in forming beliefs about the world, then, appears to presuppose P_2.

If this is right, then, on a neutral reading, B_2 can be independently justified.

5. A defensible normative argument?

Even if the above considerations are accepted, it does not automatically follow that the B_2-formulation of the normative argument is defensible. For there is another premise, (d), which might be false.

I have heard the following argument raised against (d).

> If *S* has inconsistent beliefs and EXP is valid, then, for any given *R*, *R* is a logical consequence of *S*'s beliefs. But if *R* is a logical consequence of *S*'s beliefs then, given how easily (defeasible) reasons can be obtained, *S* does indeed have (defeasible) reason to believe *R*. It follows that, for every proposition, *S* has *some* reason to believe that proposition. So (d) is false.

Call that the *weak-reason argument*.

The weak-reason argument straightforwardly fails. It assumes that EXP is valid in order to infer that *R* is a logical consequence of *S*'s beliefs. However, the validity of EXP is precisely what is at issue, so cannot legitimately be assumed.[5]

[5] Note that (d) is a *premise*, so the fact that the validity of EXP is hypothesised at (a) is irrelevant: premises must be justified independently of any hypotheses raised in the course of an argument.

To be clear, the specific wording of (d) does *not* permit the assumption that EXP is valid. Notice that (d) has the following form:

(4) Even if *P*, *Q*,

where *P* is the proposition *that* S*'s set of beliefs is inconsistent and any proposition whatsoever is entailed by it (courtesy of EXP)*; and *Q* is the proposition *that there are propositions that* S *has no reason whatsoever to believe*. Now, (4) is stated in this way to avoid the charge of begging the question; but it is in fact equivalent to (5):

(5) (If *P*, then *Q*) and (if not-*P*, then *Q*),

which is equivalent to:

(6) *Q*.

So to argue against (4), one can argue: that *P* is true and *Q* false; or that not-*P* is true and *Q* false; or simply that *Q* is false. It is insufficient to assume *P* and argue *on that assumption* that *Q* is false.

I will now use these comments to sketch an argument in favour of (d). I reiterate two points. First, I do not take the following to *establish* (d): my ultimate aim is *not* to show that the B_2-formulation is successful and that we should therefore reject EXP. I aim to provide a clear strategy for providing a potentially compelling defence of the B_2-formulation. Second, the argument in favour of (d) is available only to *non-dialetheists*. As such, one could straightforwardly resist the argument by endorsing dialetheism. However, for any theorist unwilling to countenance trivialism (the view that *every* proposition is both true and false), dialetheism would in turn lead to the rejection of EXP.

In light of the equivalence between (4) and (5), one can argue for (d) by defending, for some *S*, each of the following:

(7) If *S*'s set of beliefs is inconsistent and any proposition whatsoever is entailed by it (courtesy of EXP), then there are propositions that *S* has no reason whatsoever to believe.

(8) If there are some propositions *not* entailed by *S*'s set of beliefs (either because her belief set is consistent or because EXP is invalid), then there are propositions that *S* has no reason whatsoever to believe.

Let us consider a case in which *S* is an ordinary person, Sandy. Suppose that Sandy's beliefs are either jointly consistent, or else they are inconsistent in the following easily imaginable way: she has double-booked herself without (yet) realising it, and believes both *that she is going to the theatre on Monday night*, and *that she is not going to the theatre on Monday night*. Moreover, let us suppose there is some proposition *R*, such that, except perhaps via her contradictory beliefs and EXP: *R* is not a logical consequence of Sandy's beliefs; Sandy's beliefs do not otherwise tell in favour or against *R*; and there are no other (say) practical, epistemic or moral reasons for Sandy to entertain or form a belief about *R*. For example, perhaps Sandy has no interest in the probability of coin tosses, and no reason to have such an interest, and *R* is the proposition: *that the next coin tossed by the author of the present paper will land heads*. Or perhaps Sandy has no interest in the history of Icelandic names or presidents, nor any reason to have such interests, and *R* is the proposition: *that Sveinn Björnsson is a former President of Iceland*.

Now, consider (8). Suppose that the antecedent is true: Sandy's beliefs just happen to be jointly consistent. Then, it seems to straightforwardly follow from our description that Sandy has no reason whatsoever to believe *R*: her beliefs do not tell for or against *R*, and she has no other practical, epistemic or moral reasons to believe it. So there are propositions that Sandy has no reason whatsoever to believe. Hence (8).

Turn to (7). Suppose that the antecedent is true of Sandy: her set of beliefs is inconsistent (because of the double-booking) and any proposition whatsoever is entailed by that set (courtesy of EXP). But now, consider a proposition, *R*, such that the *only candidate (defeasible) reason* that Sandy might have to belief *R* is that it is entailed (courtesy of EXP) by her set of beliefs. As before, *R* might be *that the next coin tossed by the author of the present paper will land heads*, or *that Sveinn*

Björnsson is a former President of Iceland. *If* Sandy has a (defeasible) reason to believe *R*, *then* this reason is courtesy of EXP.

I will shortly sketch an argument that Sandy *lacks* a (defeasible) reason to believe *R*.[6] To this end, we will need the following principle:

P_3: A (defeasible) reason to believe that not-*P* is *ipso facto* a (defeasible) reason not to believe that *P*.

This principle is highly plausible *given a denial of dialetheism*. Suppose that dialetheism is false: then there are no true contradictions, and propositions of the form *P* and not-*P* cannot be simultaneously true; so the truth of not-*P* precludes the truth of *P*; in which case a (defeasible) reason to believe the former is *ipso facto* a (defeasible) reason not to believe the latter. Hence P_3.[7]

Let us sketch an argument, then, that Sandy *lacks* (defeasible) reason to believe *R*. Were Sandy to possess such a reason, it would be generated by a chain of reasoning along the following lines:

(9) Sandy is going to the theatre on Monday night.
Sandy is not going to the theatre on Monday night.
Therefore, *R*.

By parity of reasoning, the following chain would thus likewise generate an equivalent reason to believe not-*R*:

[6] By appealing to B_2, one can also argue that Sandy *does* have a reason to believe *R*. But this is irrelevant. Once the supposition of the antecedent has been discharged, such an argument could only establish that: *if S's set of beliefs is inconsistent and any proposition whatsoever is entailed by it (courtesy of EXP), then there are* no *propositions that S has no reason whatsoever to believe.* But this is consistent with (7). (That is, 'if *P* then *Q*' is consistent with 'if *P* then not-*Q*'; the conditionals jointly imply not-*P*.)

[7] The argument fails for the dialetheist as, for her, the truth of not-*P* does not preclude the truth of *P*.

(10) Sandy is going to the theatre on Monday night.

Sandy is not going to the theatre on Monday night.

Therefore, not-*R*.

But then, by P_3, (10) generates a (defeasible) reason *not* to believe *R*. That is: Sandy would have a (defeasible) reason to believe *R* only in virtue of a chain of reasoning that, by parity of reasoning, would likewise generate an equivalent (defeasible) reason not to believe *R*. Moreover, as nothing here relies on the specific details of (9) and (10), the point generalises. *For any (defeasible) reason that Sandy has to believe R, she likewise has (by parity of reason) an equivalent (defeasible) reason not to believe R.*

We now have a choice. *Either* we say that Sandy has a number of *pairs* of (defeasible) reasons—to believe *R* and not to believe *R*—that are equal and opposite, generating no *resultant* reason. (This is *not* to say that the reasons are *defeated*. Neither the reason generated by (9) nor the reason generated by (10) is undermined by the other: the reasons are in play, they merely cancel each other out.) *Or* we say that Sandy has no (defeasible) reason to believe *R*, nor (defeasible) reason not to believe *R*.

Which option should we take? There is, it seems, *no practical or epistemic difference whatsoever* between the two pictures: Sandy's overriding reasons will be *precisely* the same either way. So the phenomena do not dictate that we adopt either picture over the other. Nevertheless, in at least three respects, the latter picture is the more natural option. First, it is less cumbersome; the latter picture does not posit pairs of reasons that are superfluous to our understanding of Sandy's situation, and is in that respect more elegant. Second, relatedly, it is potentially problematic to posit reasons that can have no practical, epistemic, moral, etc., effect; their practical, epistemic, moral, etc., import is, after all, their raison d'être. It is not clear that we are justified in positing reasons that cannot influence how we ought to behave. Third, notice that in both pictures it will likely *appear* to Sandy that she has no reason whatsoever to believe *R*, and no reason whatsoever not to believe *R*. This appearance,

however, is borne out *only* in the latter picture. Thus the latter picture is less revisionary, and accords more with first-person experience.

Each of these considerations suggests that, at least to some extent, it is more natural to adopt the latter picture—on which Sandy has neither reason to believe *R* nor reason not to believe *R*. Of course, a detailed examination of these pictures would be required to provide a compelling case. But, we have at least seen that there is a clear strategy for arguing that, on present assumptions, we should adopt the latter picture.

But it follows from the latter picture that, given our description of the case, Sandy does not have any reason whatsoever to believe *R*: we have just ruled out the only candidate (defeasible) reasons. So, given that picture, there are propositions that Sandy has no reason whatsoever to believe. But that is the consequent of (7). So we have sketched an argument for the consequent of (7) from its antecedent. Hence, we have sketched an argument for (7).

Thus, we have sketched arguments for (7) and (8), and thus for (d). So, as things stand, we should conclude that the B_2-formulation of the normative argument is defensible.

6. Concluding remark

There is a defensible formulation of the normative argument against explosion. I have responded to Steinberger's objections to that formulation, and sketched defences of two key premises: the bridge principle B_2; and the claim that, even if *S*'s set of beliefs is inconsistent and any proposition whatsoever is entailed by it (courtesy of explosion), there are propositions that *S* has no reason whatsoever to believe.

While the arguments herein are not conclusive, they provide a clear strategy for defending the B_2-formulation of the normative argument, available to any non-dialetheist. In light of the recent criticisms levelled at normative arguments, this is of no little importance. Further examination is required before we can establish whether or not normative considerations tell in favour of paraconsistent logic.

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